A new principle of dynamic range expansion by analog-to-digital converting

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Abstract

A new method for the dynamic range expansion in analog-to-digital conversion is presented. It is based on none-regular sampling of the continuous signals. The sampling is performed in such regions where the signal has a small value. Samples with large values are determined by means of the solution of the equations system. A method for solving of equations system with good numerical properties is discussed.

The main characteristic of Analog Digital Convertor (ADC) is its dynamic range (DR). This characteristic affects performance, power consumption, cost and other properties of such unit. Increasing of DR is a very important target of ADC design. The goal of this paper is to find way of DR expansion, using of irregular sampling frequency.

Usually DR of a signal can be defined approximately only so ADC can limit the signal as it is shown in figure 1.

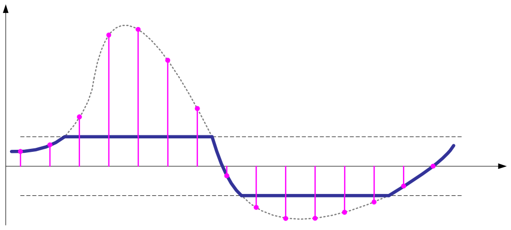

Figure 1. The signal limitation by ADC thereof its low DR

There is a way to solve this problem. For example, it is possible to get samples only at such points where signal value is less than ADC DR. Samples are exceed ADC DR can be interpolated using samples with little values. Note: because common conditions for sampling, average number of samples per time unit must be more or the same in case uniform sampling. So for irregular sampling we have to get samples more frequently in segment where the signal has value less ADC DR as it is shown on figure 2.

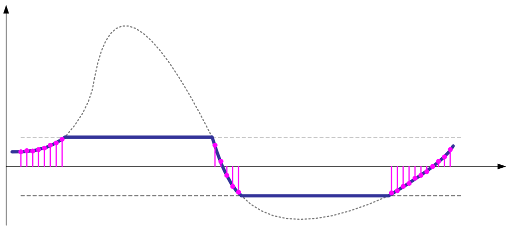

Figure 2. Irregular sampling of analog signal

Interpolation is executed following way. Usually analog signal can be represented by some basis $\varphi_n(t)$:

$$f(t)=\sum_{n=1}^{\infty} C_n\varphi_n(t).$$

But it is very difficult to work with infinitive sum so we have to transform function $f(t)$ to get finite sum.

$$\tilde{f}(t)=P[f(t)]=\sum_{n=1}^{N}\tilde{C}_n\varphi_n(t).$$

P is an operator that cuts rest coefficients. It is possible to say P is filtering in the basis $\varphi_n(t)$. The main property of this operator is the following:

$$f(0)=\tilde{f}(0).$$

Every basis has a special operator P. For basis $e^{-j\omega nt}$ it is a usual filter. For Kotelnikov basis this operator is windowing:

$$\tilde{f}(t) = f(t)w(t).$$

$w(t)$ is a usual window like Kaiser window [1], etc.

So the first stage is calculation $\tilde{f}(t)$ in points where value $f(t)$ is less than ADC DR. The result is a column. Then we calculate values of all basis functions in points $t = t_1, t_2, ..., t_N$. At last, we can find $\tilde{C}_n$ by solving equations system:

$$\begin{bmatrix} \varphi_1(\mathrm{t}_1) & \varphi_2(\mathrm{t}_1) \ ... \ \varphi_\mathrm{N}(\mathrm{t}_1) \\ \varphi_1(\mathrm{t}_2) & \varphi_2(\mathrm{t}_2) \ ... \ \varphi_\mathrm{N}(\mathrm{t}_2) \\ \multicolumn{2}{c}{\dots\dots\dots\dots\dots\dots\dots\dots} \\ \varphi_1(\mathrm{t}_\mathrm{M}) & \varphi_2(\mathrm{t}_\mathrm{M}) \ ... \varphi_\mathrm{N}(\mathrm{t}_\mathrm{M}) \end{bmatrix} \begin{bmatrix} \tilde{C}_1 \\ \tilde{C}_2 \\ ... \\ \tilde{C}_N \end{bmatrix} = \begin{bmatrix} \tilde{f}(\mathrm{t}_1) \\ \tilde{f}(\mathrm{t}_2) \\ ... \\ \tilde{f}(\mathrm{t}_\mathrm{M}) \end{bmatrix}.$$

After solving this system it is possible to calculate

$$f(0) = \tilde{f}(0) = \sum_{n=1}^{N} \tilde{C}_n \varphi_n(0).$$

Other words, finding one value over ADC DR it means to solve the equation system.

But there is a problem here. We can not to solve such system absolutely correct. Some rounding error is always. We need of guaranteed accuracy algorithm. If not, during calculation it is possible our system will become singular one.

For guarantee of solution on any processor platform it is necessary to use accuracy ε. That accuracy must be several digits less than $\dfrac{1}{\mu(A)}$, $\mu(A)$ is a condition number of the matrix:

$$\mu(A) = \|A\| \|A^{-1}\|.$$

For example, if $\mu(A) \approx 10^{10}$ then it is enough $\varepsilon \approx 10^{-16}$. If $\mu(A) \approx 10^{14}$ or 10^{15} then such matrix practically is singular.

The value of cumulative calculation error depends on used algorithm. It is not possible to estimate this error a priory. A posteriori estimation can be found this way. If terms $\tilde{f}(\mathrm{t}_\mathrm{k})$ and $\varphi_n(\mathrm{t}_\mathrm{k})$ specified exactly so it's possible use the theorem [2]:

Let C_{calc} is the calculated solution of system, A is the matrix of $\varphi_n(\mathrm{t}_\mathrm{k})$. After calculation of a misalignment $r = f - AC_{calc}$ it is possible make a following estimation:

$$\frac{\|C_{calc} - C\|}{\|C\|} \le \mu(A) \frac{\|r\|}{\|f\|}.$$

This estimation is overrated usually but there are cases when this formula becomes equality. How it works the next example shows.

Let an analog signal is known on segments $t \in [0,20] \cup [30,50]$. The segment $t \in [20,30]$ has to be interpolated by series $\sum_{k=1}^{15} C_k \sin\frac{\pi kt}{l}$, $l = 50$. Segments $[0,20]$ and $[30,50]$ are split onto 10 equal segments and now it is possible to create equation system.

$$\sum_{k=1}^{15} C_k \sin\frac{\pi k t_n}{l} = \tilde{f}(t_n),$$

$t_n = 2n$ if $n = 1,2,...,10$ and $t_n = 2n + 30$ if $n = 11,12,...,10$. In this case the condition number of the matrix is $\mu(A) = 81.4565$. We set beforehand known solution of the system $AC = f : C_k = 1$ and calculate f. Then using MATLAB we get a calculated solution. Accuracy of calculations is $\varepsilon \approx 10^{-16}$, the misalignment norm is $\|r\| \approx 10^{-15}$ so error estimation is $81.4565 \cdot 10^{-15}$.

At last, results of analog signal interpolation on platform ZSP400 are shown (figure 3). The interpolation is processed by polynomial with K-th degree

$$f(t) = \sum_{k=0}^{K} C_k t^k .$$

with solving equation system of the same order.

The axis "input sine frequency" is relation $\frac{\Omega}{\Omega_s}$ (Ω_s is the sampling frequency and Ω is a sine's frequency). The axis "level of harmonics" is nonlinear distortions that appear as a result of interpolation of the sine with frequency Ω by polynomial with number of terms from 2 to 8. Equation systems with degree from 2 to 6 are calculated with accuracy 16 bits and from 7 to 8 are calculated with accuracy 32 bits.

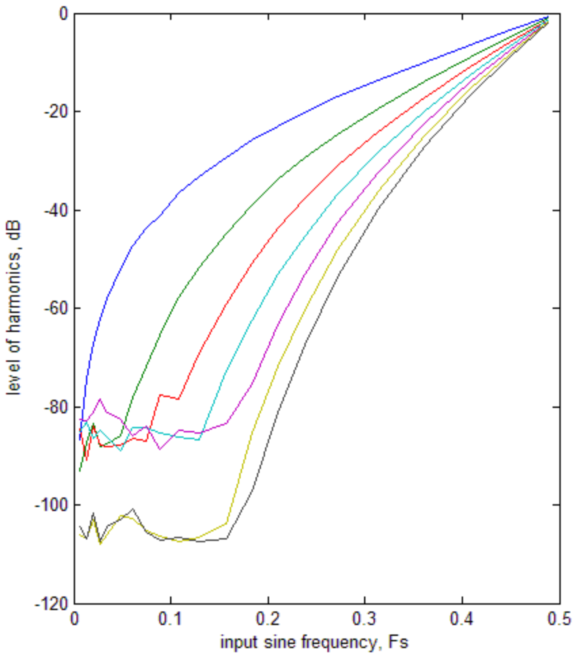

Figure 3. Interpolation by polynomial

Figure 3 shows a result of interpolation for sine only, but the same result is gotten for any signal with a limited spectrum. In that case nonlinear distortions is defined only by spectrum segment near frequency $\Omega_{\max}$ if $\Omega_{\max}$ is a maximum frequency in signal's spectrum.

[1] Y. A. Romanuk, "Fundamentals of digital signal processing", MIPT, Moscow, 2005, pp. 1-332.

[2] E. A. Biberdorf, N. I. Popova, "Guaranteed accuracy of modern algorithms of linear algebra", SBRAS, Novosibirsk, 2006, pp. 319.