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MATHEMATICAL MODEL OF ACYCLIC FUZZY CONTROL FOR TRAFFIC SIGNAL SYSTEM WITH ADAPTIVE UNCONDITIONAL TRAM PRIORITY

The mathematical model of automatic control of the traffic signals system is proposed, which implements adaptive unconditional priority for the tram approaching the intersection. The mathematical model is based on a fuzzy control law and considers the dynamics characteristics of the tram from the moment it is registered by the intersection entry detector. An example of the proposed model application in the simulation for a complex intersection is provided and a comparative analysis of the obtained time characteristics of traffic flows for fuzzy control and existing adaptive methods that do not implement the tram signal priority is performed.

1. Introduction

Public transportation (PT) or mass transit is the most effective mean of moving a large number of people according to the criterion of the required space and energy consumption, especially in densely populated regions. The introduction of high-quality PT is a significant factor in the growth of cities as centers of economic activity, because travel time reduction, as well as high reliability of the transport system, allow more participants to be involved in economic processes with lower personal costs. Efficient PT has less negative impact on the urban agglomerations' environment due to more efficient use of energy resources, as well as reducing the congestion of the cities' road network, offering an alternative way of commuting [1,2]. Therefore, the right choice of city's mobility strategy is one of the essential components that improve the quality of the urban environment. And efficient use of energy resources is an additional opportunity to significantly reduce pollution of the environment and improve transport service.

Depending on economic, technological, political, etc. factors, an appropriate degree or degrees of separation (spatial priority) of the PT infrastructure is chosen. V. Vuchik proposed the PT modes classification consisting of three (A, B, C) right of way (RoW) categories [3], decreasing the category reduces the dependence of PT mode on the road situation, but increases construction and maintenance costs of such transport infrastructure type [4]. Thus, the PT of category RoW A is completely independent of the traffic situation with its completely separated infrastructure, most often it is heavy rail (rapid transit/metro, urban railway, such as Kyiv City Express). The cost of building a RoW A transport solution from scratch limits its use to only the most important transport arteries of the agglomeration, leaving large urban areas unserved by quality inter-district connections, which is especially critical for decentralized post-industrial cities with developed service industries. Other categories of both RoW B and RoW C are implemented mainly by trams and buses (trolleybuses) but differ in traffic organization and scope. In contrast to the PT RoW C category, which provides local service, has frequent stops ($\approx 100 \div 400$ m) [4] and moves mainly in the flow of general traffic albeit can have special signals and lanes marked by line, RoW B category is less dependent on traffic conditions, as it has a spatial priority, implemented by physically separated lanes. However, only the RoW A PT is free from any at grade intersections with non-parallel traffic flows, so transit signal priority (TSP) systems are an important part of implementing a transport solution that partially or completely belongs to RoW B category.

To optimally distribute time resources between transport resources, it is necessary to perform modeling of tram dynamics, which will allow to correctly calculate the time of tram's approach to the intersection and, if possible, eliminate the need for a complete halt. This will avoid high tram starting currents, which together with the time efficiency will significantly improve its energy efficiency.

The aim of the study is to develop a mathematical model (MM) of intersection traffic signals control system, which implements the adaptive tram priority, based on tram dynamics so that it crosses the signalized intersection as quickly as possible. To achieve this goal, it is necessary to solve a number of problems:

- obtaining a mathematical model of the tram and determining the approximate time of reaching the intersection by tram;
- anticipation of the possibility of incomplete and inaccurate data being used by the model;
- adaptation to diverse types of intersections, traffic flows intensities and configurations of tram routes;
- unconditional minimization of tram passenger delay and deviation from the schedule;
- ensuring the model's stability in temporary non-standard road situations.

2. Analysis of literature sources

In case of intersection of traffic flows, the tram time priority is applied to reduce its de-lay at traffic signals [5]. If TSP is applied independently at a particular intersection, such a priority is called local. In the case of traffic signals priority control together at several intersections, the priority is considered coordinated. Traditionally, there are passive and active TSP [6]. Also, with the increase of embedded systems computing power, it becomes possible to give priority in real time depending on the traffic situation, this approach is recently referred to as adaptive priority [7]. This priority iteratively calculates the sequence of signals and their duration (traffic signal plan), based on both local characteristics of the PT and the system-wide characteristics of traffic (delays, halts, etc.). Adaptive priority requires early detection of PT vehicle to calculate the time of arrival and can be implemented on the existing system of adaptive traffic signals control, but this is not a necessary condition for its implementation [7]. Typically, the control model of traffic signal system with adaptive priority for PT interacts with the following components:

- means of detection (and interaction with) PT vehicle;
- traffic detection system;
- priority request generator (s) and servers, control system with traffic signals plans in real-time.

The model itself implements a signal control algorithm, considering the impact on other traffic and ensuring the safety of pedestrians. By definition, a TSP-capable traffic signal will not adversely affect the coordination of traffic signals [8].

Most modern systems that implement TSP work in a coordinated manner as part of the urban traffic control (UTC) systems in real time. Existing control systems with real-time TSP can be divided into two categories [9]:

- with a constant length of traffic signal cycle (rule-based) [10,11], time parameters are gradually adapted to fluctuations in traffic conditions in real time;
- with variable cycle length (optimization-based) [12-14], adaptive commands constantly optimize the traffic signal plan using the rolling horizon method [15].

It should be noted that the European approach to the implementation of TSP is quite severe, with higher levels of priority and less attention to possible negative impacts on other traffic [16].

As the process of adaptive control of the traffic light system with priority for general traffic, and the tram in particular, cannot be based on perfect, complete data, the appropriate approach to this issue is soft computing, which seeks to adhere to the principle of tolerance for imprecision, uncertainty, partial truth, and approximation to achieve tractability, robustness and low solution cost [17]. Existing studies mainly consider the application of fuzzy computing to adaptive traffic signal control in general, without a detailed focus on TSP [18-20]. In addition, existing methods that explicitly implement TSP using soft computing use a constant duration of the traffic signal cycle and/or predetermined traffic light stages (phases), which inevitably imposes limitations on adaptive control [21,22]. For the problem of adaptive priority, the most common are methods of evolutionary and fuzzy computing.

Since the description of traffic is naturally associated with imprecision, and traffic control at a particular intersection is conducted according to certain rules, fuzzy control is a natural approach to solving this problem. The application of fuzzy logic to the problem of traffic light control involves control based on expert knowledge, rather than modeling a directly controlled process [23], which reduces computational costs. Nevertheless, the listed above papers do not use the data of PT vehicle dynamics, as there authors mainly consider a PT of RoW C category. Therefore, it is necessary to apply a mathematical model of the longitudinal dynamics of the tram approaching the intersection, slowing down for safety reasons.

3. Mathematical model of tram dynamics

The involved tram MM [24] (Tatra T3) determines its deceleration or acceleration forces by calculating the torque that the wheels receive from the motor, which determines the effect on the linear dynamics of the tram. Modeling of the braking process involves the use of an electrodynamic braking system applied at speeds > 5 km/h. The main characteristics of the tram car that determine its dynamics are given in Table 1 [25]. The calculation of the current weight of the tram uses the assumption that the average weight of one passenger is 65 kg.

Table 1

Characteristics	Notation	Value
Estimated wheel radius	r	325 mm
Wheel mass	m_{wh}	195 kg
Power of traction motors	P_{max}	4×50 kW
Maximum speed of an empty tram	–	65 $km \cdot h^{-1}$
Average deceleration during service braking	–	1.4 $m \cdot s^{-2}$
Mass of empty tram car	–	18.1×10^3 kg
Nominal passenger capacity of the tram car	–	100

The linear dynamics of the tram consists of three forces. The first is the adhesion force, which depends on the tram velocity:

$$F_{ad}(v_s) = \mu(v_s)Mg \quad , \tag{1}$$

where $\mu(v_s) = c_{ad}e^{-b_{ad}v_s}$ is adhesion coefficient; $v_s = r\omega_{wh}(t) - v(t)$ is wheel slip velocity; $\omega(t)$ is angular velocity of a tram wheel rotation; $v(t)$ is linear velocity of the tram $\left[ms^{-1}\right]$; M is current mass of the tram; a_{ad}, b_{ad}, c_{ad}, d_{ad} are adhesion parameters (Table 2) [26], depending on weather conditions (0 means ideal conditions, and 3 are the worst).

Table 2

Parameter	Adhesion			
	0	1	2	3
a_{ad}	0.54	0.54	0.54	0.05
b_{ad}	1.2	1.2	1.2	0.5
c_{ad}	1.0	0.2	0.1	0.08
d_{ad}	1.0	0.2	0.1	0.08

The second force is the rolling resistance, which is determined by the empirical formula [27]:

$$F_r(v) = A + Bv + Cv^2 , \tag{2}$$

where A is coefficient associated with the mass acting on the axle of the tram, combines the frictional resistance of the rail and wheel, resistance due to track defects, as well as friction of bearings; B is coefficient associated with the lateral displacements of the tram, due to which there is a friction force between the flange of the wheel and the inner part of the rail; C is coefficient related to the cross-sectional area of rail vehicle, as well as space in-between vehicle, so its impact becomes more noticeable at speeds above 80 km/h. For the involved model A=0.0147M, B=125.83, C=0.

The third force is a projection of the force of gravity and depends on the slope (gradient) of the track $\theta : F_s(\theta) = Mg\sin(\theta)$.

Linear dynamics is determined by the angular velocity of the tram wheel, which is affected by motor torque and adhesion force torque. The torque of the tram motor depends on the position of the notch $pos \in \{-7,...,0,...,7\}$. The authors of the model [24] experimentally identified the dependence of motor torque on the notch position:

$$\tilde{\tau}_{mot}(pos) = \begin{cases} K_p pos & \text{if } pos > 0 \text{ and } \omega\tilde{\tau}_{mot} < P_{\max}, \\ P_{\max}/\omega & \text{if } pos > 0 \text{ and } \omega\tilde{\tau}_{mot} \ge P_{\max}, \\ K_n pos & \text{else,} \end{cases} \tag{3}$$

where K_p, K_n are experimentally determined proportionality coefficients corresponding to the maximum and minimum acceleration at the highest and lowest position of the notch, respectively (K_p=1449, K_n=1176).

The transition to time space τ_{mot} is using transfer function $H(s) = 3/s + 3$.

So, for the tram model involved, the dynamics equations are:

$$\dot{v} = \frac{F_{ad} - F_r - F_s}{M}, \qquad \dot{\omega} = \frac{\tau_{mot} - \tau_{ad}}{J}, \tag{4}$$

where $J = 0,5 m_{wh} r^2$ is the tram wheel moment of inertia.

4. Mathematical model of fuzzy control of traffic signals system

During the process of controlling the traffic light system, it is necessary to ensure the coordination of signals for different directions of movement. The considered systems and methods of traffic signal control system with adaptive TSP use two approaches:

- fixed, based on traffic signal stages, combining several compatible signals for different directions. Signals included in certain phases are determined in advance;
- flexible, based on signal groups (SG) [28]. Signals decomposition allows to adapt the composition of stages in real time, considering the current state of traffic and interoperability of SG (Fig. 1). By determining the start time and duration for the group, and not for each signal, a significant savings in computing resources can be achieved.

ID	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
0	0	2	1	2	2	0	1	0	2	1	1	0	2	1	0	0	0	0	1	0	0	0	0	0
1	5	0	5	2	2	2	2	0	0	2	2	2	3	0	2	0	0	0	0	2	0	0	2	2
2	1	2	0	2	2	1	1	0	2	0	1	2	2	0	0	1	0	0	0	0	0	1	0	0
3	5	3	5	0	0	5	0	0	3	0	0	2	2	0	0	0	0	2	0	0	0	0	0	3
4	5	3	5	0	0	5	0	0	3	0	5	2	0	0	0	0	0	3	0	0	0	0	3	3
5	0	2	1	2	2	0	0	0	2	0	1	0	2	1	0	0	0	0	1	0	0	0	0	0
6	1	2	1	0	0	0	0	0	2	0	1	2	0	0	0	0	0	0	1	0	0	1	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	0	0	0
8	5	0	5	3	3	5	5	0	0	5	5	3	3	0	0	0	2	0	0	0	2	0	0	2
9	1	2	0	0	0	0	0	0	2	0	0	2	0	0	0	0	0	0	1	0	0	1	0	0
10	1	2	1	2	2	1	1	0	2	0	0	2	2	1	0	0	0	0	0	0	0	1	0	0
11	0	3	5	0	2	0	5	0	3	5	5	0	0	0	0	0	0	0	0	3	0	0	0	3
12	5	4	5	2	0	5	0	0	4	0	5	0	0	0	0	0	2	0	0	0	0	0	0	2
13	8	0	0	0	0	8	0	0	0	0	8	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	5	0	0	0	6	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	4	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	8	0	0	0	0	8	8	0	0	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	4	0	0	0	0	0	5	0	0	0	6	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	5	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	8	0	0	0	8	0	0	8	8	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	4	0	0	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	4	0	5	5	0	0	0	5	0	0	6	6	0	0	0	0	0	0	0	0	0	0	0

Fig. 1. Conflict matrix (in seconds)

Figure 2 describes the diagram of the proposed traffic signal control model. Three types of traffic light signals are considered, which control the movement of distinct categories of road users: trams *y1(t)*, pedestrians *y2(t)* and general traffic *y3(t)*. The initial state of the signals of the traffic signals system is determined by the parameter *g(t)*. In the general case, the control input *s(t)* to the

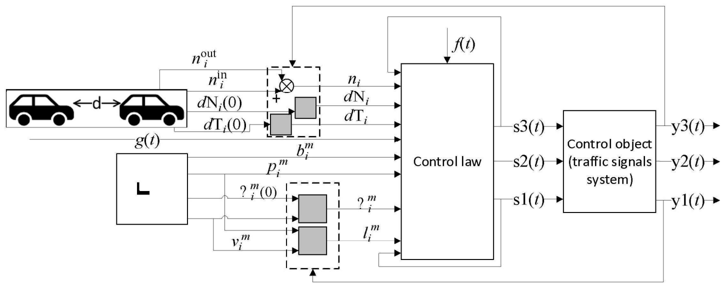

Fig. 2. Diagram of the adaptive control system of intersection traffic signals with tram priority

traffic signals system is a set of SG identifiers $\bigcup s_i$, which are combined into the stage S_j. The control law actively takes into account the characteristics of traffic and tram(s) for each s_i. The characteristics of the car flow are: n^{in} is the number of vehicles that have passed through the entry detector; n^{out} is the number of vehicles that passed through the exit detector at the stop line; *dN(0)* is initial time distance between vehicles on the entry detector; *dT(0)* is initial time distance between vehicles on the stop line detector. The characteristics of the tram *m* are: *b* is time elapsed from detection by entry detector until detection by exit detector located after the intersection; *p* is passenger occupancy; $\delta(0)$ is cumulative deviation from the schedule at the time of detection by the entry detector; *v* is current speed. The application of traffic signals control affects these characteristics, which are then analyzed by fuzzy rules of the control law. The control law provides for passive consideration of accidental disturbance in the form of a pedestrian, which briefly affects the dynamics of a tram.

The only characteristic that determines the order of SG activation is their weights or importances w, which are determined at the second level of the model [29] (Fig. 3). For SG, which control the general traffic, the weight $w(s3_i)$ is determined depending on the number of vehicles between the detectors [29], herewith $\exists w(s1_i) > 0 \mid \forall w(s3_i) \triangleq 0.25 \times w(s3_i)$. To determine the weight of tram

SG, in this paper we propose the rules of fuzzy inference (Table 3), which take into account the passenger occupancy and schedule adherence (Fig. 4). Stages S_j are determined using the conflict matrix CM:

$$s_i \in S_j : S_{a=k} \cap \left(\bigcup_{b \neq k} S_b \right) = \varnothing; \forall s_x, s_y \in S_j, CM[x,y] = 0 . \tag{5}$$

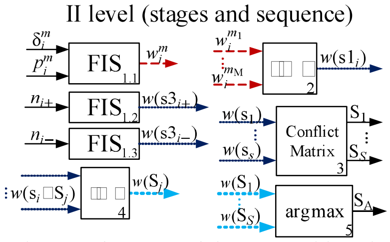

Fig. 3. Diagram of the second level of the model

Table 3

d_shed	*n_pass*			
	zero	a few	medium	many
ahead	zero	low	medium	high
on time	zero	medium	high	highest
behind	medium	high	high	highest

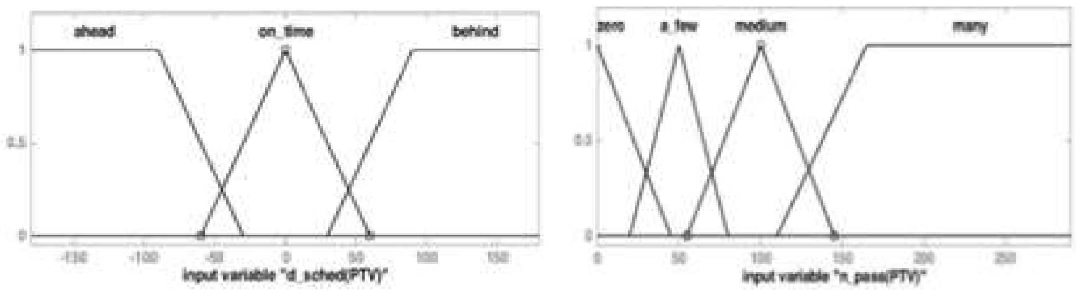

Fig. 4. Membership functions of fuzzy variables *d_shed* and *n_pass*

Next stage $S_A = agr \max_j \left(\left\| \sum_{s_i \in S_j} w[s_i] \right\| \right)$.

The first level of the model (Fig. 5) determines the SG time parameters for the composed phases. The rules for determining the action on a green SG $s3_{i+}$ are defined as in the FUSICO project [29]. For the problem of tram priority, the control rules for several stages [23] have been extended to take into account the tram SG importance $w(s1_i)$ (Fig. 6). The actions of GE (green extension), GR (green recall) are further analyzed fuzzy rules (Fig. 7) taking into account the tram dynamics, where the confidence degree in the action is determined, similar to the rule base for SG $s3_{i+}$.

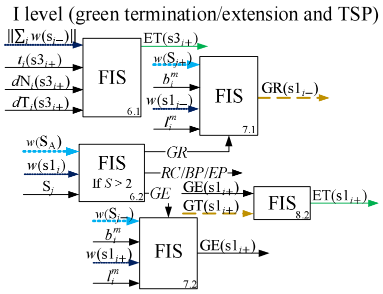

Fig. 5. Diagram of the first level of the model

Variable *l(PTV)* is a relative value of the time the tram needs to reach the intersection, which depends only on the dynamics of the tram and is invariant to the time since detection:

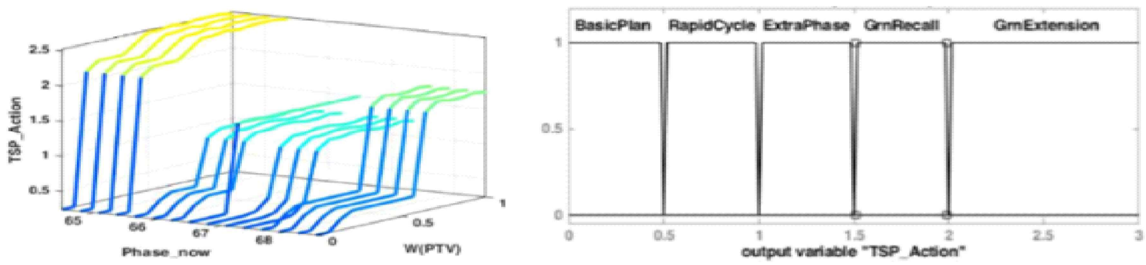

Fig. 6. Reaction function for several stages and membership function of output crisp variable

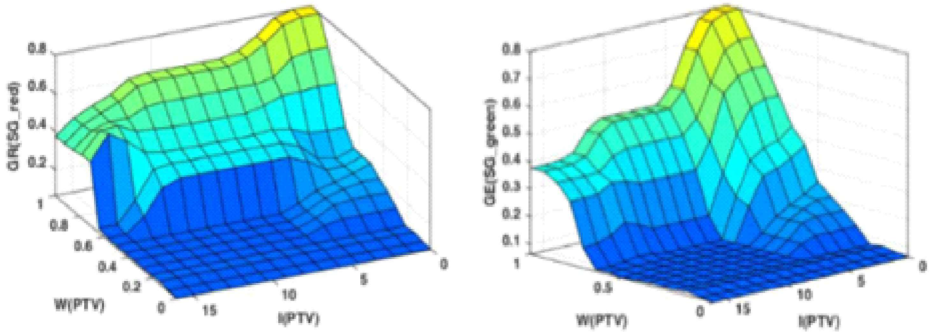

Fig. 7. Reaction functions to determine action on SG $s1_{i-}$ та $s1_{i+}$

$$l(PTV) = l_i^m - \frac{s_{\det}}{v_{\max}} + b_i^m, \tag{6}$$

where l_i^m is time until the tram reaches the intersection, obtained by modeling its linear dynamics; $s_{\det}$ is distance between the entry point of the detector and the boundary of the intersection (100 m); $v_{\max}$ is the maximum speed allowed before the intersection.

Table 4

$GE(s1_{i+})$	$GT(s1_{i+})$				
	Wait	RWait	NSure	RTer	Term
Wait	Term	Term	Term	Term	Term
RathWait	RTer	RTer	RTer	Term	Term
NotSure	NSure	NSure	NSure	RTer	Term
RatherExt	RExt	RExt	RExt	RExt	RExt
Extend	Extnd	Extnd	Extnd	Extnd	Extnd

To resolve potential conflicts between priority requests from the inhibiting SG $s1_{i-}$ during active $s1_{i+}$ relevant rules have been added (Table 4). Also, for the correct service order of priority requests, during the composition of stages, in the SG $s1_{i-}$, allowing movement from the common lane, the value b_i^m is analyzed.

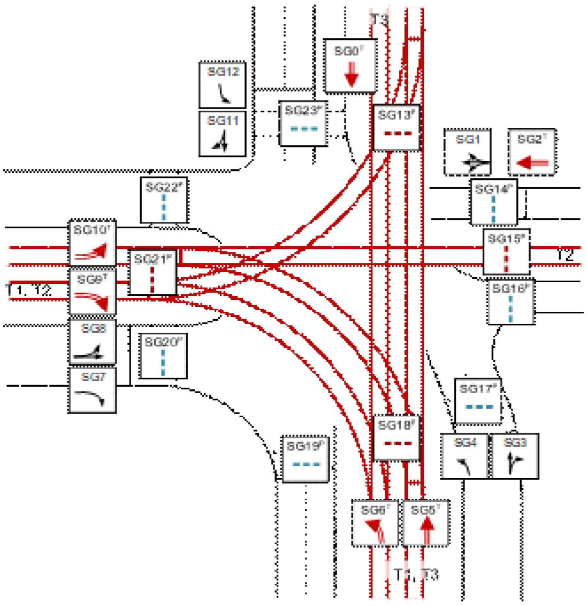

Fig. 8. Layout of the created intersection

5. Analysis of modeling results

The modeling was performed for an artificial intersection with tram lines branching and intersections of adjacent directions for trams and general traffic, which complicates the task of optimal adaptive control with a fixed stages approach (Fig. 8).

SUMO tool was utilized for modeling [30], in which reference control methods for in-tersection traffic signals system are available: gap-based or vehicle-actuated (VA) and delay-based (DB). These methods are based on a fixed predefined sequence of stages and adjust the duration of the permissive signal depending on the time interval between consecutive vehicles for the VA method and the delay of detected vehicles depending on their speed in the range of lane detectors for the DB method.

The study of the applicability of the developed method fuzPrioPro for traffic control is performed for non-peak and peak conditions of transport demand. Taking into account the geometric features of the intersection, the following demand parameters are set for general traffic in non-peak conditions (Table 5).

For peak conditions, the value of the traffic intensity increases 1.5 times. The tram is characterized by deterministic demand and has a defined schedule with intervals. Expected passenger traffic is determined in passengers per hour per direction, p/h/d. For non-peak and peak conditions, the parameters of passenger flow for the specified tram lines are given in Tables 6 and 7 respectively.

The measured characteristics of vehicles traffic flows that accumulate in front of traffic signals are their number. For trams of each direction of each line, the analyzed metrics are time loss and number of halts. The modeling is performed for 3600 s, with a step of 3 s for the control method. The period of averaging of data on the general traffic is 60 s, for trams data the period is 30 s. Passenger occupancy of tram sets is determined by the normal distribution with a standard deviation $\sigma = 20$ and the mean value of μ according to defined parameters (Table 6, 7).

Table 5

Flow origin	Intensity, veh/h	Flow direction probability, %			
		N	E	S	W
North	103	—	11	53	36
East	80	7	—	41	52
South	240	8	17	—	75
West	180	18	10	72	—

Table 6

Line	Origin	Occupancy, pas/tram	Avg trams per tram set	Intensity, sets/h
T1	West	77	1.3	13
	South	78	1.4	15
T2	West	40	1.0	7
	East	65	1.0	9
T3	North	73	1.1	10
	South	80	1.2	11

Table 7

Line	Origin	Occupancy, pas/tram	Avg trams per tram set	Intensity, sets/h
T1	West	100	2.0	16
	South	115	2.0	18
T2	West	90	1.0	10
	East	100	1.0	10
T3	North	93	1.5	13
	South	94	1.7	14

Figure 9 shows the delays of trams of lines T1 and T3 (SG 5, 6), as well as the number of complete halts within the detectors range in off-peak conditions. Compared to the DB and VA methods, the proposed method with absolute tram priority significantly reduces the mean delay of both the tram vehicles and all their passengers. In addition, there is a lower standard deviation of tram delays on both lines, as well as a lower mean value of the number of halts when using a fuzzy control method.

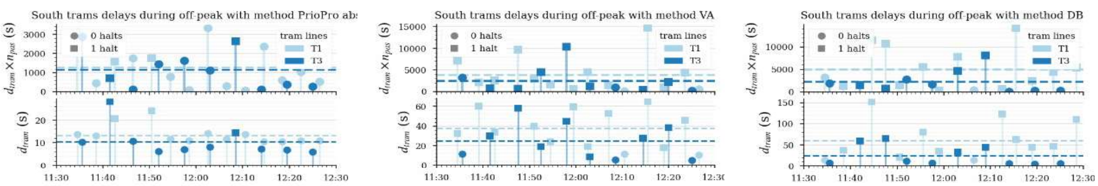

Fig. 9. Comparison of time characteristics of trams from the south in non-peak conditions

A comparable situation is observed for trams of lines T1 and T2 (SG 9, 10) (Fig. 10). It can be noted that the average delay of T2 line's passengers is lower despite the higher mean delay of the

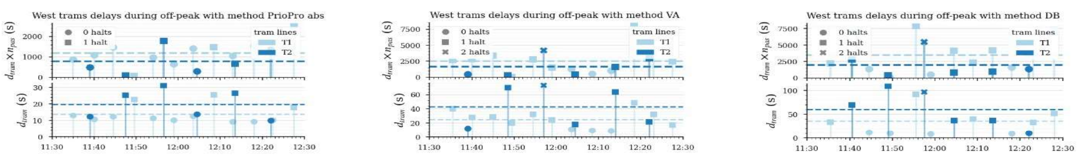

Fig. 10. Comparison of time characteristics of trams from the west in non-peak conditions

trams themselves, which is due to its lower passenger flow. For the PrioPro method there are no cases with a double halt at the intersection. For other methods, this occurs approximately at the same time for the tram line T2, for which there is no possibility of conflict-free crossing with the approach of a fixed sequence of stages (Fig. 8) (conflict with trams of the T2 line in counter direction).

Indeed, analyzing the delays of trams from the east, one can see the passage of the tram, which is likely to be the cause of several halts (Fig. 11). The following Table 8 generalizes characteristics of tram traffic in non-peak conditions.

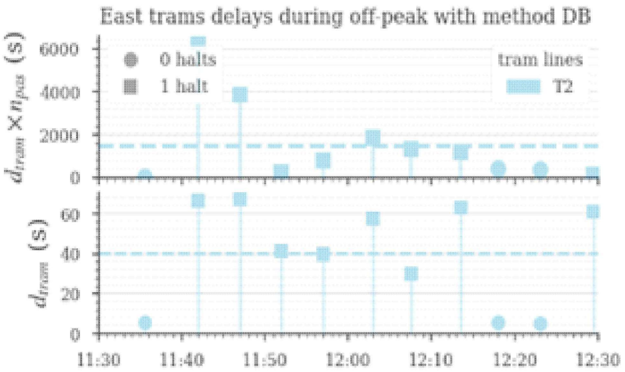

Fig. 11. The drawback of the fixed stages approach for complex intersections

Table 8

line	TramTimeLoss		PasTimeLoss		HaltsPerTram		Speed		method
	mean	std	mean	std	mean	std	mean	std	
T1	48.79	37.0	4291.4	3790.16	0.77	0.43	12.62	6.57	DB
T2	47.18	30.86	1668.47	1875.29	0.82	0.53	13.51	8.82	DB
T3	22.77	23.1	2108.37	2446.82	0.42	0.51	21.14	9.04	DB
T1	32.06	17.43	3212.27	3359.43	0.81	0.4	14.48	5.36	VA
T2	36.03	22.61	1495.24	1484.26	0.88	0.49	14.48	7.11	VA
T3	27.54	17.86	2730.65	3073.45	0.74	0.45	17.3	7.88	VA
T1	13.54	4.6	1232.67	868.34	0.19	0.4	21.21	2.83	PrioPro
T2	12.87	7.95	486.03	478.16	0.24	0.44	22.85	5.49	PrioPro

One can see consistently lower time delays for all tram lines under PrioPro control. Compared to the DB method, the expected delay of tram vehicles was reduced by 67%, while against the VA method reduction is by 62%. If we compare the delay of passengers, the biggest gain is observed for the T1 line and reaches 71% compared to the DB method. The VA method has less difference, for all lines it is about 65%. In terms of the number of halts, their expected value is consistently lower for the PrioPro method. The mean speed of the passage and regularity also increased.

Analyzing the characteristics of the general traffic flow, it can be stated that the pro-posed method does not have a significant negative impact on traffic in non-peak condi-tions. It is seen that the VA method leaves the stationary mode in the middle of modeling, yet, in the end, the queue begins to discharge (Fig. 12).

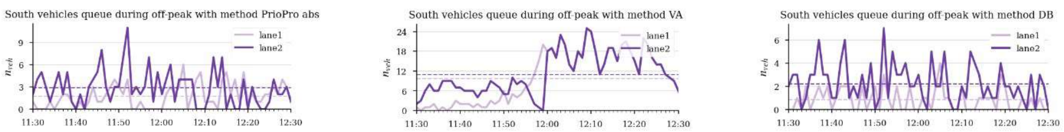

Fig. 12. Comparison of time characteristics of general traffic from the south under non-peak conditions

Table 9

origin	lane	nVehWithin		method	nVehWithin		method	nVehWithin		method
		mean	std		mean	std		mean	std	
S	1	0.85	0.97	DA	9.73	7.9	VA	1.75	1.57	PrioPro
S	2	2.23	1.69	DA	10.84	6.6	VA	2.87	2.35	PrioPro
W	1	0.76	1.0	DA	0.62	0.75	VA	0.52	0.85	PrioPro
W	2	0.65	0.89	DA	0.72	0.84	VA	0.52	0.81	PrioPro

The following Table 9 is a summary of the queue length on the busiest approaches to the intersection in non-peak conditions. The

PrioPro method is slightly inferior to the DB method in throughput for the southern approach but is best for the traffic flow from the west.

Compared to off-peak conditions, the delay of trams from the south varies even less, especially for the T3 line. However, under fuzzy control there is a case of double tram halt (Fig. 13).

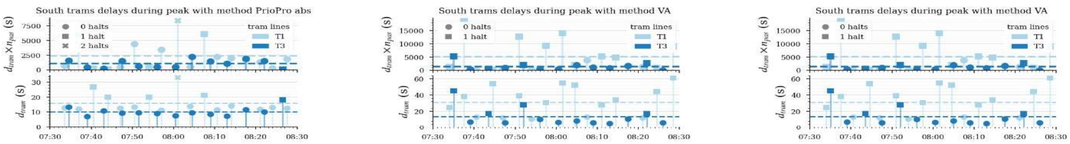

Fig. 13. Comparison of time characteristics of trams from the south under peak conditions

Table 10 shows the generalized results of modeling the trams flow under saturated conditions of traffic. Compared with the DB method, the average reduction in both types of delays is 80%, for the VA method is less: 57%. The number of halts has significantly decreased, and reduction is observed even in comparison with non-peak conditions. The expected value of the passage speed has also increased.

Table 10

line	TramTimeLoss		PasTimeLoss		HaltsPerTram		Speed		method
	mean	std	mean	std	mean	std	mean	std	
T1	73.68	50.3	14425.16	16986.18	0.75	0.44	10.85	7.51	DB
T2	82.18	46.87	10913.72	11201.96	1.14	0.36	7.91	4.29	DB
T3	41.98	35.68	4254.85	6065.89	0.64	0.49	16.05	8.82	DB
T1	34.85	18.36	6761.53	7769.66	0.72	0.46	14.55	6.0	VA
T2	34.45	21.07	4363.26	5478.01	0.67	0.48	15.05	7.66	VA
T3	18.34	16.82	2032.48	2863.9	0.48	0.51	21.92	6.98	VA
T1	15.26	5.79	2885.98	3274.74	0.28	0.58	20.8	2.89	PrioPro
T2	10.97	4.2	1338.95	1244.44	0.14	0.36	23.43	3.68	PrioPro
T3	10.22	3.39	1095.28	956.78	0.08	0.28	25.04	2.5	PrioPro

As for the general traffic, under saturated conditions, all control methods at some mo-ment go into non-stationary mode, but stabilize relatively quickly, except for the VA method (Fig. 14).

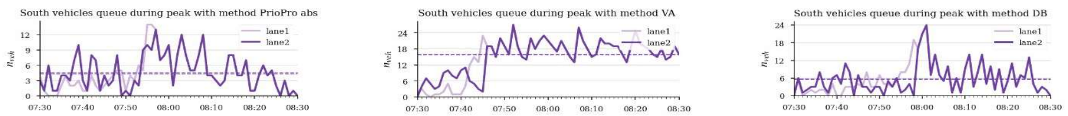

Fig. 14. Comparison of time characteristics of general traffic from the south under peak conditions

Analyzing modeling results for general traffic under peak conditions (Table 11), one can see that the proposed PrioPro method provides stable control for the busiest traffic directions.

Table 11

origin	lane	nVehWithin		method	nVehWithin		method	nVehWithin		method
		mean	std		mean	std		mean	std	
S	1	5.79	5.37	DA	15.78	7.04	VA	4.21	3.61	PrioPro
S	2	5.56	5.11	DA	15.88	6.45	VA	4.49	3.47	PrioPro
W	1	1.66	1.72	DA	1.0	1.48	VA	1.08	1.69	PrioPro
W	2	1.79	1.94	DA	2.1	2.46	VA	3.0	2.55	PrioPro

6. Conclusions

The proposed mathematical model of acyclic traffic signal control system with uncon-ditional priority for the tram as a category RoW B public transportation, considering the dynamics of its movement as it approaches the intersection, significantly reduces passen-ger delay, and allows to avoid complete tram halt. At the same time, there is a slight deterioration in general traffic flow, which is expected with the application of absolute transit signal priority.

A further direction of research may be to expand the rules for dealing with the case of interruption of an active signal group for general traffic by priority request from waiting tram for more smooth traffic flows control.

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