Mathematical Model of Plasma Space for Electronic Technologies

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Abstract **- The paper is devoted to studying the plasma used in technologies of the electronic industry. It gives the characteristic of plasma space on the basis of a system of Maxwell-Boltzmann equations. Solving these equations is represented in the form of Fourier transformation and Green functions. Fluctuation-dissipative theorem and method of Longevin sources for calculating electric filed fluctuations are used.**

Key words**: electric field, correlation functions, fluctuation-dissipative theorem, temperature.**

INTRODUCTION

In studying plasma systems researching electromagnetic properties is of great importance. Statistical theory (ST) of thermodynamic equilibrium is based upon the use of fluctuation-dissipative theorem. Thus, the problem of finding correlation functions of an electric field is reduced to calculating the function of the system response to external disturbance. One of the ST approaches is the method of Langevin sources. At that it is necessary to derive correlation functions and equations of the distribution of the fluctuations of the electric field. The present paper is devoted to developing a statistical model of a semi-infinite plasma being abut with a dielectric. Such a model is used in astrophysics, nuclear fusion, solid-state plasma of semiconductors, plasma spraying of thin films and gas-discharge plasma. The first section of the paper considers the problem of exciting electric waves in a dielectric on the basis on Maxwell equations and linearized kinetic equation with a collision integral in an $\vec{r}$ - approximation. The solution of these equations is represented in the form of Fourier expansion. The second section uses the fluctuation-dissipative theorem for finding the correlation functions of a electric field which are expressed in terms of the linear response function. The distribution of Green functions along z-axis has been found. The third section makes use of the Langevin approach to finding the correlation functions of the electric field.

Manuscript received August 14, 2014.

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I. DISTRIBUTION OF ELECTRIC FIELD

Let us consider a homogeneous quasineutral plasma system taking up the semifinite space $(x, y < \infty, z > 0)$. The exterior domain is filled with a dielectric of permittivity $\tilde{\varepsilon}(\omega)$. We shall get equations for calculating the distribution of the fluctuation electric field established by arbitrarily distributed induced sources $\vec{J}(\vec{r};t)$ and $\tilde{\vec{J}}^e(\vec{r},t)$. The sought distributions can be found as a result of the joint solution of the Maxwell equations for the exterior domain and linearized system of Maxwell-Boltzmann equations for the plasma occupied domain [1]

$$rot\vec{B}(\vec{r},\omega) = -i\frac{\omega\varepsilon}{c}\vec{E}(\vec{r},\omega) + 4\pi\left(\frac{\sum_\sigma \vec{J}(\vec{r},\omega) + \vec{J}^e(\vec{r},\omega)}{c}\right)\times$$
$$\times\left\{-i\omega + \upsilon\frac{\partial}{\partial\vec{r}} + \frac{e_\sigma}{m_\sigma}\left(\vec{E} + \frac{v\vec{B}}{c}\right)\frac{\partial}{\partial v}\right\}\times\delta f_\sigma(\vec{r},v,\omega) +$$
$$+\frac{e_\sigma}{m_\sigma}\left\{\vec{E}(\vec{r},\omega) + \frac{v\vec{B}(\vec{r},\omega)}{c}\right\}\times\frac{\partial f(v)}{\partial v} = L\delta f_\sigma(\vec{r},v,\omega);$$

$$rot\vec{E}(\vec{r},\omega) = i\frac{\omega}{c}\vec{B}(\vec{r},\omega), \tag{1}$$

where $\vec{J}(\vec{r},\omega)$ is the induced current of particles having charge e_σ, mass m_σ and density n_σ; $\varepsilon(\omega)$ is the plasma permittivity; $\delta f_\sigma(\vec{r},v,\omega)$ is the deviation of particle distribution; $\vec{E}$, $\vec{B}$ are electric and magnetic fields; $L = -v$ is the linearized operator of particle collisions; v is the frequency of particle collisions; “~” is the exterior domain value. Let us consider the model of mirror reflection of charged particles from the interface region [2]. To harmonically analyze the sources $\vec{J}(\vec{r},\omega)$, $\tilde{\vec{J}}^e(\vec{r},\omega)$, we shall make Fourier transform. After a transition from variables $\vec{r}$ to variables $\vec{k}$, we shall obtain [3-5]

$$A(\vec{r},\omega) = \frac{1}{(2\pi)^3}\int_{-\infty}^{\infty} dk \exp(i\vec{k}\vec{r})A(\vec{k},\omega), \tag{2}$$

and the sought Fourier-components of an electric field can be represented in the form (Fig. 1)

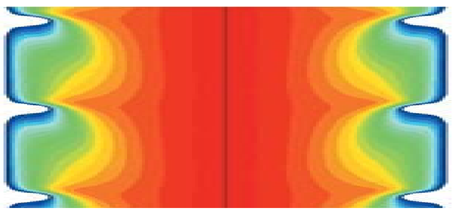
$$\begin{cases} \vec{E}(\vec{k},\omega) = -i\dfrac{4\pi}{\omega\Lambda(\vec{k},\omega)} \times \left\{ \vec{J}^{e}(\vec{k},\omega) + \dfrac{c\vec{E}^{e}(\vec{k},\omega)}{2\pi S(\vec{k},\omega)} \right\}; \\ \tilde{\vec{E}}(\vec{k},\omega) = -i\dfrac{4\pi}{\omega\tilde{\Lambda}(\vec{k},\omega)} \times \left\{ \tilde{\vec{J}}^{e}(\vec{k},\omega) - \dfrac{c\tilde{\vec{E}}^{e}(\vec{k},\omega)}{2\pi S(\vec{k},\omega)} \right\}. \end{cases} \quad (3)$$

$$\Lambda(\vec{k},\omega) = \varepsilon(\vec{k},\omega) - \left(\delta - \frac{\vec{k}}{\vec{k}^{2}} \right) \frac{c^{2}\vec{k}^{2}}{\omega^{2}};$$

$$\tilde{\Lambda}(\vec{k},\omega) = \left(\delta - \frac{\vec{k}}{\vec{k}^{2}} \right) \tilde{\vec{\Lambda}}_{T}(\vec{k},\omega);$$

$$S(\vec{k},\omega) = \frac{i}{\pi}\frac{c}{\omega}\int_{-\infty}^{\infty} d\vec{k}\,\tilde{\Lambda}^{-1}(\vec{k},\omega);$$

$\varepsilon(\vec{k},\omega)$ is the permittivity tensor.



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THE FOLLOWING X,Y,Z VALUES ARE IN GLOBAL
COORDINATES
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02	-1327.7	-1445.9	3655.1	4414.7
04	-1379.3	-1459.7	3543.9	4959.4
06	-1394.4	-1313.6	3627.4	4208.4
08	-1389.7	-1337.4	3425.5	3967.5
10	-1395.9	-1349.9	3246.7	3956.6
12	-1356.7	-1394.3	3276.8	3897.1
14	-1482.8	-1334.8	3284.8	3437.8
16	-1367.2	-1369.2	3524.6	3986.5
18	-1354.3	-1365.5	3412.6	3954.8
20	-1334.8	-1324.8	3434.3	3934.9
22	-1382.6	-1386.7	3487.8	3782.8
24	-1327.8	-1393.7	3482.3	3527.9
26	-1325.7	-1375.4	3385.5	3265.7
28	-1306.8	-1327.7	3575.2	3816.8
30	-1358.9	-1349.2	3476.9	3527.6
32	-1380.5	-1381.8	3554.7	3789.5
34	-1359.7	-1368.7	3417.5	3596.8
36	-1307.3	-1383.5	3747.6	3879.5
38	-1320.7	-1392.2	3834.8	3930.7
40	-1306.4	-1374.8	3752.2	3659.9
42	-1357.6	-1346.9	3974.8	3549.3
44	-1325.9	-1329.5	3841.4	3529.3
46	-1427.8	-1364.8	3862.2	3527.8
48	-1456.4	-1247.4	3879.7	3407.9
50	-1145.3	-1268.3	3843.4	3594.7

Fig. 1. Electric fluctuations in plasma

II. Mathematical Model of a Thermodynamical Equilibrium System

If a plasma is in the state of ST thermodynamic ecvilibrium, the correlation function has the form [6] in line with the fluctuation-dissipative theorem

$$\vec{E}(\vec{r})\dot{\vec{E}}(\vec{r}) = -\theta(\omega,T) \times (G(\vec{r},\vec{r},\omega) + \dot{G}(\vec{r},\vec{r},\omega)), \quad (4)$$

where the average energy of a quantum harmonic oscillator

$$\theta(\omega,T) = \frac{\hbar\omega}{2} + \frac{\hbar\omega}{\exp\left(\dfrac{\hbar\omega}{T}\right) - 1},$$

$G(\vec{r},\vec{r},\omega)$ is the function of linear system response to the external disturbance.

As the system is homogeneous in $\vec{r}$, we shall pass over to variables $\vec{k}$.

$$\vec{E}(z)\dot{\vec{E}}(\dot{z}) = -\theta(\omega,T) \times (G(\vec{k},z,\dot{z},\omega) + \dot{G}(\vec{k},z,\dot{z},\omega)). \quad (5)$$

Let us find the distribution of Green functions of the Maxwell-Boltzmann system along the axis z

$$\begin{cases} G(\vec{k},z,\dot{z},\omega) = \dfrac{1}{(2\pi)^{2}} \int_{-\infty}^{\infty} d\vec{k} \int_{-\infty}^{\infty} d\dot{\vec{k}} \exp(i(\vec{k}z - \dot{\vec{k}}\dot{z}))G(\vec{k},\dot{\vec{k}},\omega); \\ \tilde{G}(\vec{k},z,\dot{z},\omega) = \dfrac{1}{(2\pi)^{2}} \int_{-\infty}^{\infty} d\vec{k} \int_{-\infty}^{\infty} d\dot{\vec{k}} \exp(i(\vec{k}z - \dot{\vec{k}}\dot{z}))\tilde{G}(\vec{k},\dot{\vec{k}},\omega). \end{cases} \quad (6)$$

We shall obtain [7] (Fig.2) substituting the Fourier - components of Green functions into the system (6)

$$\begin{cases} \vec{E}(\vec{k},\omega) = \dfrac{1}{2\pi} \int_{-\infty}^{\infty} d\dot{\vec{k}} G(\vec{k},\dot{\vec{k}},\omega)\vec{J}(\dot{\vec{k}}); \vec{J}(\vec{r},t) = \tilde{\vec{J}}^{e}(\vec{r},t); \\ \tilde{\vec{E}}(\vec{k},\omega) = \dfrac{1}{2\pi} \int_{-\infty}^{\infty} d\dot{\vec{k}} \tilde{G}(\vec{k},\dot{\vec{k}},\omega)\vec{J}(\dot{\vec{k}}); \tilde{\vec{J}}^{e}(\vec{r},t) = 0; \\ \vec{E}(\vec{k})\dot{\vec{E}}(\dot{\vec{k}}) = -\theta(\omega,T) \times \left(G(\vec{k},\dot{\vec{k}},\omega) + \dot{G}(\vec{k},\dot{\vec{k}},\omega) \right); \\ \tilde{\vec{E}}(\vec{k})\dot{\tilde{\vec{E}}}(\dot{\vec{k}}) = -\theta(\omega,T) \times \left(\tilde{G}(\vec{k},\dot{\vec{k}},\omega) + \dot{\tilde{G}}(\vec{k},\dot{\vec{k}},\omega) \right). \end{cases}$$

III. Using the Method of Longevin Approach

The contribution to the correlation functions of electric fields is taking into account the heat radiation of external medium. The sources $\vec{J}^{e}(r,\omega)$, $\tilde{\vec{J}}^{e}(r,\omega)$ given in various space areas can be considered independent. Using the system of equations (3) of the problem of exciting by arbitrarily distributed sources, we will get the following equations [8]

$$\vec{E}(\vec{k})\dot{\vec{E}}(\dot{\vec{k}}) = \left(\frac{4\pi}{\omega} \right)^{2} \Lambda^{-1}(\vec{k},\omega)\dot{\Lambda}^{-1}(\dot{\vec{k}},\omega) \times$$

$$\times \left\{ \vec{J}^{e}(\vec{k})\dot{\vec{J}}^{e}(\dot{\vec{k}}) + \left(\frac{c}{2\pi} \right)^{2} \times \frac{\vec{E}^{e}\dot{\vec{E}}^{e} + \tilde{\vec{E}}^{e}\dot{\tilde{\vec{E}}}^{e}}{S(\vec{k},\omega)\dot{S}(\vec{k},\omega)} + \frac{c\vec{J}^{e}(\vec{k})\dot{\vec{E}}^{e}(\vec{k},\omega)}{2\pi\dot{S}(\vec{k},\omega)} + \frac{\vec{E}^{e}(\vec{k},\omega)\dot{\vec{J}}^{e}(\dot{\vec{k}})}{S(\vec{k},\omega)} \right\}; \quad (8)$$

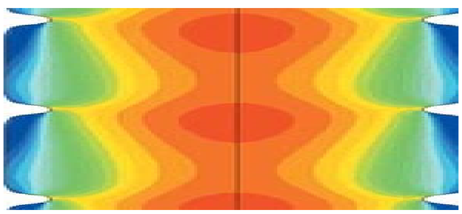
$$\vec{\tilde{E}}(\vec{k})\dot{\vec{\tilde{E}}}(\dot{\vec{k}})=\left(\frac{4\pi}{\omega}\right)^2\tilde{\Lambda}^{-1}(\vec{k},\omega)\dot{\tilde{\Lambda}}^{-1}(\dot{\vec{k}},\omega)\times$$

$$\times\left\{\left(\vec{\tilde{J}}^e(\vec{k})\dot{\vec{\tilde{J}}}^e(\dot{\vec{k}})\right)+\left(\frac{c}{2\pi}\right)^2\times\frac{\vec{E}^e\dot{\vec{E}}^e+\vec{\tilde{E}}^e\dot{\vec{\tilde{E}}}^e}{S(\vec{k},\omega)\dot{S}(\vec{k},\omega)}-\frac{c\vec{\tilde{J}}^e(\vec{k})\dot{\vec{\tilde{E}}}^e(\vec{k},\omega)}{2\pi\dot{S}(\vec{k},\omega)}+\frac{\vec{\tilde{E}}^e(\vec{k},\omega)\dot{\vec{\tilde{J}}}^e(\dot{\vec{k}})}{S(\vec{k},\omega)}\right\}. \tag{9}$$

Setting Langevin sources in the external area to zero can result in erroneous results. When $T=\tilde{T}$, equations (8) and (9) are reduced to equations for the correlation function of electric field Fourier-components. Introducing complimentary sources enables to calculate the distribution of an electric field in plasma. The equation for the energy of heat radiation from unit of plasma surface to the external area is determined by a normal component of the Umov-Pointing vector [9] (Fig. 2)

$$P(\omega)d\omega=\int_{\theta\le 0.5\pi} d\Omega\cos\theta I(\omega,\theta,T,\tilde{T})d\omega, \tag{10}$$

where $I(\omega,\theta,T,\tilde{T})=I(\omega,\theta,T)-I(\omega,\theta,\tilde{T})$ is the intensity of heat radiation per unit of solid angle $d\Omega=\sin\theta d\theta d\varphi$; θ is the vectorial angle from the outer normal to the plasma boundary.



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POST1 NODAL TEMPERATURE INTENSITY LISTING
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TIME= 1.0000 LOAD CASE= 0
THE FOLLOWING X,Y,Z VALUES ARE IN GLOBAL COORDINATES

NODE	EX	EY	EZ	ESUM
01	2529.8	2732.3	3672.8	5745.5
02	2585.4	2743.5	3655.1	5714.7
04	2339.9	2854.3	3543.9	5959.4
06	2394.5	2313.6	3627.4	5887.8
08	2379.7	2437.4	3625.5	5968.5
10	2495.9	2449.4	3646.7	5954.9
12	2456.6	2594.8	3676.6	6043.2
14	2582.3	2534.7	3684.3	6037.3
16	2567.2	2569.4	3687.6	6087.5
18	2564.3	2575.5	3712.6	6054.8
20	2634.5	2584.8	3734.3	6097.9
22	2682.6	2684.3	3789.4	6099.3
24	2627.8	2639.7	3782.3	6027.9
26	2525.7	2575.5	3685.5	5965.7
28	2576.2	2527.6	3575.2	5976.5
30	2558.3	2549.3	3586.5	5927.4
32	2575.3	2551.4	3554.7	5909.4
34	2589.7	2568.5	3527.6	5896.8
36	2577.2	2683.8	3570.5	5879.4
38	2520.7	2692.2	3435.8	5793.7
40	2486.4	2574.4	3452.2	5759.5
42	2457.3	2546.8	3445.8	5749.5
44	2425.9	2529.5	3438.4	5729.8
46	2427.8	2564.8	3439.2	5707.5
48	2457.4	2547.7	3494.7	5695.9
50	2428.2	2528.3	3443.5	5653.7

Fig. 2. Temperature distribution in plasma

The actuality of this scientific paper consists in the fact that its results are being used at Kharkov National University of Radio Electronics at accomplishing Ukraine's Ministry of Education and Science (UMES) theme № 269 "SOLAR" for manufacturing thin films. Using the methods developed allows technological indices to be changed: oxygen concentration temperature at constant discharge power of 2.05 W/m^2 and chamber pressure of 5×10^{-3} mega bars. It is found that glow - discharge plasma exerts a considerable energetic and heat effect on the film in the course of magnetron sputtering. It permits to produce films without special substrate heating.

IV. Conclusion

The paper is of practical use as it has solved the problem of exciting a plasma system under thermodynamic equilibrium on the basis of a charged particle mirror reflection model. A method of developing plasma ST is suggested which takes into account external medium radiation. This method is based on the use of Langevin approach when random sources of fluctuations are introduced in plasma and external areas. To calculate the electric field the values of the Green functions of the Maxwell-Boltzmann equations system have been found and their harmonic analysis has been done. The results obtained on the base of the fluctuation-dissipative theorem and Langevin approach within the same temperatures of plasma and external medium are equivalent. The work has been done on UMES scientific theme №0107U002295.

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